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Modeling Exchange Market Volatility Risk in Rwanda Using GARCH-EVT Approach

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Abstract: **This study has two main parts, part one consist of highlighting the statistical procedure which can be used to find out the best model among GARCH-family models for modelling the exchange market of Rwanda using the daily exchange rate data for 2038 days since 2010/01/01. Part two consist of modeling residuals extracted from a selected GARCH model using the Generalized Pareto Distribution (GPD) and the Generalized Extreme Values (GEV), then estimate the Value at Risk(VaR) and the Expected Shortfall (ES) from both GPD and GEV models. In the first part, the study employed four models from the family of Generalized Autoregressive Conditional Heteroscedasticity models (GARCH) combined with the mean model ARMA; among which this study chose to estimate ARMA(1,1)-GARCH (1,1), ARMA(1,1)-GJR-GARCH(1,1), ARMA(1,1)-EGARCH(1,1) and ARMA(1,1)-APARCH(1,1,1) and choose the best among them, with a view to approximate the dynamics exchange market volatility. The estimation results reveal ARMA(1,1)-APARCH (1,1,1) to be the most appropriate specification for modeling the Rwandan exchange market. The diagnostic testing of residuals from the ARMA(1,1)-APARCH(1,1,1) reveals that there is no ARCH effect in residuals, there is no serial autocorrelation in residuals but that residuals are not normally distributed and have fat- tail. Finally we estimate σ_{t+1} and μ_{t+1}.In the second part, the study extract residuals from ARMA(1,1)-APARCH(1,1,1), and use the two models Generalized Pareto Distribution (GPD) and Generalized Extreme Values (GEV) in modeling these residuals . The study shows that our series fit both GPD and GEV models and estimate VaR (Value At Risk) and ES (Expected Shortfall). Finally the estimated σ_{t+1} and μ_{t+1} from the ARMA (1,1)-APARCH(1,1,1) and both VaR and ES are used to estimate $VaR(Z)_q$ and $ES(Z)_q$.**

Keywords: **Volatility, GARCH-family models, Extreme value theory, Value at risk, Expected shortfall.**

1. INTRODUCTION

In these two last decades the world have been characterized by financial crisis, these includes the Asian crisis of 1997, the Russian debts crisis of 1998, the crisis followed by the fall of world trade center in 2001 and other many periods of crisis till nowadays. Since then many developed countries have as priority to improve the way of managing risks for their financial institutions due to these surprises turbulence in the worldwide financial market nowadays. These periods of economic crisis have affected all African countries include Rwanda in many ways especially through the banking crisis of 2008 because of the dependence of Rwanda financial system on banking sector.

Due to these unexpected significant fluctuations in these few past years, many economists have questioned the existence of a good risk management methodology to solve these financial instabilities, This is why a huge effort has invested into developing statistical method to protect financial systems against unpredictable fluctuations and losses, since then many researches demonstrated that extreme value theory (EVT) can be successfully applied in financial market to predict the risk for a financial system the theory started in McNeil(1999). The EVT refers to the branch of statistics which deals with the extreme deviations from the mean of a probability distribution. EVT in finance deals with tails of a distribution to evaluate the highest loss of a financial market with certain probability over a certain time horizon, as is applied to event with a very low probability of occurrence. This thesis shows how we can model use GARCH-EVT approach to model the two risk measures VaR (value at risk) and ES (expected shortfall) for a short term forecasting. Banks, especially, have a responsibility to maintain proper risk management practices, since they hold the majority of the population's money. Value-at-risk and expected shortfall has been implemented by the Basel Committee on Banking Supervision as a regulatory method for financial institutions to estimate the market risk associated with their outstanding assets (Basel II, 2006). The VaR refers to the amount risked over some period of time with a fixed probability, since VaR is considered as the measure of tail risk it shows the degree of sensitivity to the financial market loss; In practice it provide a loss threshold exceeded with some small predefined probability usually 1% and 5%, in other word it shows the maximum loss that can't be exceeded with a certain level of confidence.

Many researches include the work of McNeil (1999), show that VaR goes with the volatility of returns, if the volatility changes through a period of time then VaR also changes through that period of time). This study will use GARCH family models to estimate the value at risk; we combine an ARMA model for modeling the dynamic conditional mean and one of the GARCH-family models for modeling the dynamic conditional volatility as GARCH, EGARCH and GJR-GARCH process and finally we choose the tail distribution that fit our model between normal distribution, student-t distribution and generalized error distribution (GED). In this study we use the exchange rate between Rwandan francs and the US dollar, daily data from the central bank in Rwanda to estimate the well fitted model among the GARCH-family models (GARCH, EGARCH and TGARCH) with a chosen tail distribution, and then we use the GARCH tail to model VaR and ES for a short term forecasting.

Many studies on exchange rate volatility suggest that there is a strong relationship between the exchange rate volatility of a currency and the economy of a country through its impact on the trade performance. Many researchers say that exchange rate volatility affects the trade flow negatively, studies like Cushman (1983, 1986) and Kenen and Rodric (1986) showed that the effect of exchange rate volatility is negative and significant. But there some research which suggest that there is a positive effect of the exchange rate on trade and that the trade performance is not sensitive to the exchange rate. The other way in which exchange rate affects an economic performance of a country is through its impact on prices. A direct effect occurs through its impact on import prices which also goes up to the consumer price. This model will help the Rwandan central bank to control the price stability and the evolution of money supply and also to manage the extreme risk that can shake our financial system.

2. LITTERATURE REVIEW

The ARCH model was first introduced by Engle (1982) for capturing time variant variance exhibited by almost all financial time series and many economic time series. The generalized version of ARCH model (GARCH model) which gives more parsimonious results than ARCH model was formulated by Bolleslev (1986) and Nelson (1990). The two other GARCH-family model that allow for asymmetric shocks to volatility are GJR-GARCH (Glosten-Jagannathan-Runkle GARCH) model introduced by Glosten et al. (1993) and EGARCH(exponential GARCH) model proposed by Nelson(1991).

The use of extreme value theory has become popular in finance, after its publication in some papers such as Embrechts et al (1999), Bensalah (2000) and Brodin and Klüppelberg (2006) and the results showed that extreme value theory methods fit the tails of heavy-tailed financial time series better than more conventional distributional approaches and that it was the best approach in estimating the tail of a loss distribution. In 1990s due to the currency crisis, stock market turbulence and credit default many research studies such as Gilli and Këllezi (2006),Mancini and Trojani (2010), showed the potential of EVT approach in finance and illustrated EVT using block maxima method(BMM) and peak over threshold (POT) in modeling VaR, ES and return level. Result showed that POT was considered to be more efficient in modeling limited data and not depending on the requirement for large data set as BMM because it exploits better the information in sampling. This study is based on the findings of Mc Neil (1999) when he worked in the combination of Extremes Value Theory (EVT) and stochastic volatility models. in this he suggested two steps in combining EVT and GARCH , The first step is to filter returns volatility fitting a GARCH model using ML (Maximum Likelihood). The second step is to apply the extreme value theory to residuals extracted from a select GARCH model using GPD (Generalized Pareto Distribution) or GEV (Generalized Extreme Values).

Mc Neil and Frey (2000) and Gencay et al (2003) tried to use the extreme value theory (EVT) to solve the problem. Contrary to VaR approaches, EVT is used to model the behaviour of maxima or minima in a series (the tail of the distribution). They used volatility and fat tail of conditional return distribution and they estimated VaR and ES by combining GARCH to estimate current volatility and use EVT for estimating the tail of the distribution. Nystöm and Skoglund (2002) also calculated VaR and ES by applying ARMA-GARCH with EVT approach to estimate extreme quantiles of univariate portfolio risk factor.

Some papers worked on the exchange rate volatility using the Rwandan exchange rate such as Ntawihebasenga et al (2014) wrote on estimating risk in Rwanda exchange rate and shows that both returns and residuals have fat tail behaviour, which shows that using GARCH-EVT approach can work very well. W.Tibesigwa and W.Kaberuka (2012) worked on exchange rate volatility of the Rwandan francs and shows that Rwanda exchange rate is highly volatile and is affected by news. W.Tibesigwa et al (2014) volatility analysis of exchange rate of emerging economies: a case of east Africa community, and showed that the existence of high rates of exchange volatility could be explained by the fact that these currencies are not pegged to any major international currency.

3. METHODOLOGY

3.1 modeling volatility using GARCH-family models:

In this study we use GARCH-family models in modeling the dynamic of the exchange rate volatility. The GARCH models to be considered are: ARMA(1,1)-GARCH (1,1), ARMA(1,1)-EGARCH(1,1), ARMA(1,1)GJR-GARCH(1,1) and ARMA(1,1)-APARCH(1,1). The best model among these three models is selected and its residuals are used in modeling the tail behaviour of our series.

3.1.1 Symmetric GARCH-model:

Contrary to ARCH-model GARCH model requires few parameters to adequately describe volatility process of an asset return. Follows GARCH (p, q) model, the variance equation is given by the following formula;

$$\sigma_t^2 = c_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_j \sigma_{t-1}^2 \tag{1}$$

Where α_i and β_j are respectively ARCH and GARCH term and c_0 is a constant; σ_t^2 and σ_{t-1}^2 are respectively the fitted conditional variance from the model and its previous value. ε_t^2 are the squared terms in the model. Its simplest form is GARCH (1,1) with the equation variance of form:

$$\begin{cases} X_t = \mu + \varepsilon_t = \mu + \sigma_t Z_t \\ \sigma_t^2 = c_0 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{cases} \tag{2}$$

This equation requires that $\alpha \geq o$ and $\beta \geq 0$; if $\alpha + \beta = 1$ the process under consideration is stationary. In this study we only consider the model with lower order GARCH (1, 1).

GJR-GARCH model:

Glosten-Jagannathan-Runkle GARCH(1,1) it allows for asymmetry effects in volatility modeling which is used to handle leverage effects. Its simplest form is written as follow:

$$\alpha_t^2 = c_0 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma h_{t-1} \varepsilon_{t-1}^2 \tag{3}$$

Where h_{t-1} is an indicator function that takes the value of 1 if $\varepsilon_{t-1} \leq 0$ and 0 otherwise; γ is the asymmetric parameter and c_0, α and β are defined as in equation (1).

EGARCH model:

EGARCH model is the extension of GARCH model that allows to efficiently capturing volatility clustering and asymmetric effect. It allows the asymmetric effect between positive and negative shocks. Its variance equation is of the form:

$$\ln\left(\sigma_t^2\right) = \alpha_0 + \sum_{i=1}^{p}\left(\alpha_i \frac{\left|\varepsilon_{t-i}\right|}{\left|\sigma_{t-i}\right|} + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right) + \sum_{i=1}^{q}\beta_j \ln\left(\sigma_{t-1}^2\right) \tag{4}$$

Where $\varepsilon_{t-1} = \frac{a_{t-1}}{\sigma_{t-1}}$ are the error terms at lag 1. a_{t-1} is the white noise series at lag 1. And γ_i is the leverage effect and is assumed to be negative in real application. In this study we use the simplest form of EGARCH (means Of order (1,1)) which can be written as:

$$\ln\left(\sigma_t^2\right) = c_0 + \alpha\left(\frac{\left|\varepsilon_{t-1}\right|}{\left|\sigma_{t-1}\right|}\right) + \gamma\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + \beta \ln\left(\sigma_{t-1}^2\right) \tag{5}$$

APARCH model:

in financial time series ,large negative returns appear to increase volatility more than positive returns of the same magnitude, this is called the “leverage effect”. APARCH(asymmetric power ARCH) is used to model better the leverage effect than the standard GARCH.

The variance equation of the APARCH (p,q) model can be written as

$$\sigma_t^\delta = c_0 + \sum_{i=1}^{p}\alpha_i(\left|a_{t-1}\right| - \gamma_i a_{t-1})^\delta + \sum_{j=1}^{q}\beta_j \sigma_{t-j}^\delta \tag{6}$$

Where $\delta > 0$ and $-1 < \gamma_j < 1, \quad j = 1,..., p$. Note that $\delta = 2$ and $\gamma_1 = ... = \gamma_p = 0$ the APARCH model is reduced to GARCH model.

3.2 EXTREME RISK MODELING:

3.2.1The generalized extreme distribution:

Consider X_n a series of iid random variable with $X_1, X_2, X_3,..., X_n$ cumulative distribution function (cdf) $F(x)$ with a stochastic maximum $M_n = \max(X_1, X_2, X_3,..., X_n)$. When dealing with financial risk $X_t = -r_t$ means negative return at time t. The cumulative distribution function of M_n is given by:

$$P(M_n \le x) = P(X_1 \le x, X_2 \le x,..., X_n \le x) = \prod_{t=1}^{n} P(X_t \le x) = F^n(x) \tag{7}$$

Considering only unbounded random variable X_t, means $F(x) < 1$, for all $x < \infty$, it holds that $F^n(x) \to 0$ for all x, if $n \to \infty$ and hence $M_n \xrightarrow{P} \infty$. M_n has to be standardized to achieve a non-degenerate behaviour limit. **Fisher-tippet theorem:** if X_n a series of iid random variables and if for a non-degenerate distribution function H , there exist a constant $c_n > 0$ and $d_n \in \Re$, then:

$$\frac{M_n - d_n}{c_n} \xrightarrow{d} H \tag{8}$$

and H Belong to a GEV distribution with the distribution function given by:

$$H_{\xi}(x)=\begin{cases} e^{-(1+\xi x)^{\frac{-1}{\xi}}}, & if \quad \xi \neq \\ e^{-e^{-x}}, & if \ \xi = 0 \end{cases} \tag{9}$$

Here is such that $1+\xi x>0$. We obtain the parameter family by defining $H_{\xi,\mu,\sigma}(x)=H_{\xi}((x-\mu)/\sigma)$ for location parameter $\mu \in \Re$ and a scale parameter of the GEV distribution and H_{ξ} gives the type of the distribution. The generalized extreme value distribution (GEV) representation three distributions depending on the shape parameter:

1: Fréchet (for $\xi > 0$):

$$\Phi_{\alpha}(x)=\begin{pmatrix} 0, & x \leq 0, \alpha > 0 \\ e^{-x^{-\alpha}} & x > 0 \end{pmatrix} \tag{10}$$

2. Gumbel ($\xi = 0$):

$$\Lambda(x)=e^{-e^{-x}}, \quad x \in \Re \tag{11}$$

3. Weibull ($\xi < 0$)

$$\Psi_{\alpha}(x)=\begin{cases} e^{-(-x)^{\alpha}} & x \leq 0, \alpha > 0 \\ 1 & x > 0 \end{cases} \tag{12}$$

The following two figures shows the probability distribution function (pdf) cumulative distribution function(cdf) of the three generalized extreme value distributions Fréchet , Gumbel and Weibull .

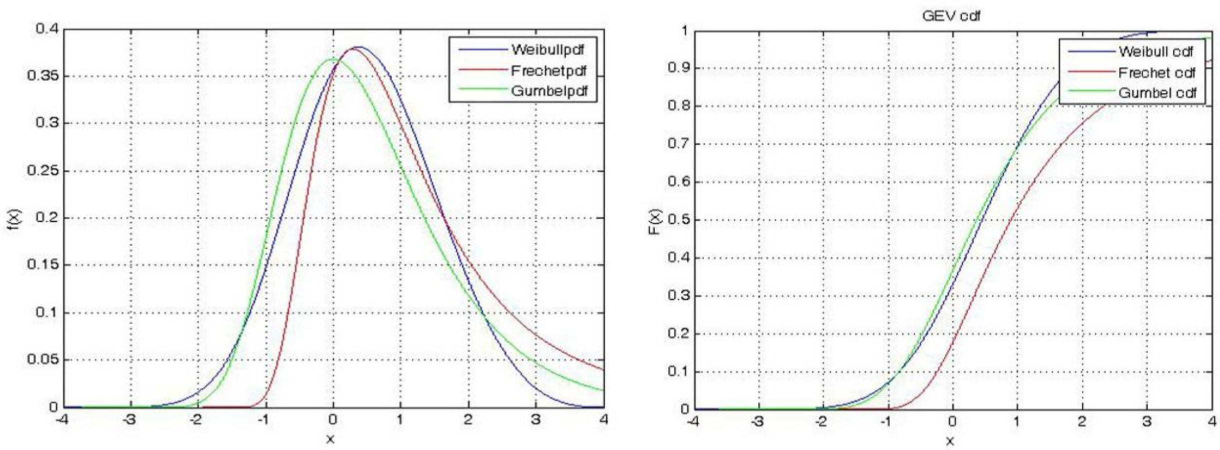

Figure 1: pdf and cdf of Weibull, Fréchet and Gumbel

Block maxima method (BMM):

The block maxima approach in extreme value theory (EVT), consists of dividing the observation period into n non-overlapping periods of equal size and restricts attention to the maximum observation in each period. For some large value of n if the data is a series of iid variable, and then the series will generate block maxima $M_{n,1}, M_{n,2}, \ldots, M_{n,m}$ (m block) which fit the GEV distribution, and the Generalized extreme value distribution fitted to the block maxima (minima) in this study is used to analyze extreme losses.

Value at risk is given by:

$$VaR(\alpha)=\mu+\frac{\sigma_t}{\xi_t}\left\{\left(-(\ln(\alpha))^{-\xi}\right)-1\right\} \tag{13}$$

Where ξ and σ_t are respectively the estimated shape parameter and scale parameter by the GEV (Generalized Extreme Value) model, and μ the estimated mean.

3.2.2 GPD and Peak Over Threshold (PTO):

The extreme value theory has two results: the use the block maxima model which fit the generalized extreme value distribution and the peak over threshold (POT) which fit the Generalized Pareto Distribution (GPD). The advantage of POT over BMM model is that in PTO all data which exceed the fixed threshold are used while in BMM they use only the maximum data in a block is selected.

Pickands-Balkema theorem: given a large class of distribution function F_u tend to fit the GPD for an increasing threshold u . Then:

$$F_u(y) \approx G_{\xi,\sigma}(y), \qquad u \to \infty \tag{14}$$

Where $G_{\xi,\sigma}$ is the Generalized Pareto Distribution (GPD) which is given by the equation.

$$G_{\xi,\sigma}(y) = \begin{cases} \left(1 - \dfrac{\xi}{\sigma} y\right)^{-\frac{1}{\xi}} & if \quad \xi \neq 0 \\ 1 - e^{-\frac{y}{\sigma}} & if \quad \xi = 0 \end{cases} \tag{15}$$

Where ξ is the shape parameter and σ is the scale parameter for the GPD (Generalized Pareto Distribution).For $y \in [0, (x_F - u)]$ if $\xi \geq 0$ and $y \in \left[0, -\dfrac{\sigma}{\xi}\right]$ if $\xi < 0$.

VaR and Expected Shortfall in GPD

Here the VaR and ES are the functions of estimated parameter of the GPD. If F is an extreme distribution with the right endpoint X_F , we can assume that for some threshold u

Then the $F_u(x) = G_{\xi,\sigma}(x)$ where $0 \leq x < x_F - u$ and $\xi \in \Re$ and $\sigma > 0$. If $x \geq u$, then:

$$\begin{aligned} \overline{F}(x) &= P(X > u)P(X > x \setminus X > u) \\ &= \overline{F}(u)P(X - u > x - u \setminus X > u) \\ &= \overline{F}(u)\overline{F}(x - u) \\ &= \overline{F}(u)(1 + \xi \frac{x - u}{\sigma}) \end{aligned} \tag{16}$$

Given $\overline{F}(u)$, $\overline{F}(x)$ is the formula for tail probabilities, its inverse gives the highest quantile of the distribution which represent the value at risk VaR and is given by:

$$VaR_\alpha = q_\alpha(F) = u + \frac{\sigma}{\xi}\left(\left(\frac{1-\alpha}{\overline{F}(u)}\right)^{-\xi} - 1\right) \tag{17}$$

For $\xi < 1$, the expected shortfall ES is given by:

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 q_x(F)dx = \frac{VaR_\alpha}{1-\xi} + \frac{\sigma - \xi u}{1-\xi} \quad (18)$$

If n is the total observation and N_u the number of observations above the threshold u. If replace F_u with $G_{\xi,\sigma}(x)$ and $F(u)$ by $(n - N_u)/n$, the tail distribution is estimated by:

$$\hat{F}(x) = 1 - \frac{N_u}{n}(1 + \frac{\hat{\xi}}{\hat{\sigma}}(x-u))^{-1/\hat{\xi}} \quad (19)$$

The inverse of the estimator for the tail distribution with a probability p gives the estimator of parameters $\hat{\sigma}$ and $\hat{\xi}$ which are used in estimating VaR and ES, by replacing them respectively in equation (2.22) and (2.23). And the ES with the probability p is given by:

$$ES_p = \frac{VaR_p}{1-\xi} + \frac{\sigma - \xi u}{1-\xi} \quad (20)$$

3.2.3 Combining GARCH with EVT models:

Applying EVT to standardized residuals (white noise) Z_t is the best way for modeling tails, as it doesn't matter the distribution of $F(z)$. Applying the EVT to the random variable X_t is an appropriate way because X_t is not independently and identically distributed. The EVT approach does not assume $F(z)$ to follow a normal distribution as it applies the POT estimation procedure to residuals. Many studies showed that in financial data is better when estimating the VaR and ES to use some measures of current volatility. If we follow the GARCH (1, 1) model the one day ahead forecast of VaR and ES are given by:

$$VaR_q^t = \mu_{t+1} + \sigma_{t+1}VaR(Z_q) \quad (21)$$

$$ES_q = \mu_{t+1} + \sigma_{t+1}ES(Z_q) \quad (22)$$

Where μ_{t+1} and σ_{t+1} are respectively the conditional GARCH estimates of mean and volatility.

This study is proceeded in three steps:

1. We choose the best model among three of the GARCH-family models combined with ARMA model (i.e. ARMA-GARCH, ARMA-TGARCH and ARMA-EGARCH) that well fit the exchange rate volatility in Rwanda using the maximum likelihood estimation. This model helps to extract residuals and estimate μ_{t+1} and σ_{t+1} using the fitted model.

2. We separate positive and negative residuals to create a new variable that contains positive residuals

3. Use the extreme value theory (EVT) to model the tail behaviour of residuals and calculate $VaR(Z)_q$ using the Generalized Pareto Distribution (GPD) and Generalized Extreme Value (GEV) tail estimation procedure; then we calculate VaR_q^t using the expression described in Equation (21).

4. RESULT AND DISCUSSION

The fact that our series is not stationary at level, in this study we work with the" log returns" or simply the log price return of 1USD in RWF denoted by " LPUSD" which is given by:

$$LPUSD_t = r_t = \log\left(\frac{y_t}{y_{t-1}}\right) \quad (23)$$

The graph for LPUSD is shown in figure 4.

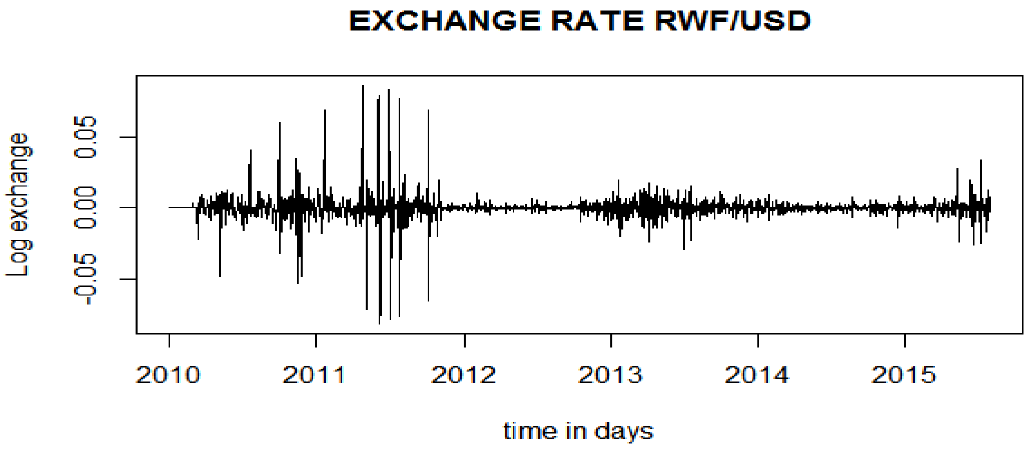

Figure 2. The graph for the LPUSD (log price of one USD)

The Figure 4 shows that our series is stationary in mean and in variance. The study used also the Augmented Dickey Fuller (ADF), the result showed that the series is stationary .the figure 2 shows the presence of the volatility clustering and the period of high volatility tend to be followed by another period of low volatility in long interval of time. These signs indicate that the use of ARCH/GARCH family models is possible. The study performed the ARCH-test to check the heteroscedasticity in our series, and results revealed the presence of ARCH-effect in LPUSD series.One of the objective of this study is to check among GARCH-family models combine with a mean equation of ARMA and choose which one can be used in modeling the exchange market of RWF/USD. In this family we will deal only with the ARMA(1,1)-GARCH (1, 1),ARMA(1,1)-GJRGARCH(1,1) ,ARMA(1,1)-APARCH(1,1,1) and ARMA(1,1)- EGARCH (1,1).

4.1 Estimation of GARCH-family models:

4.1.1. Estimation of ARMA(1,1)-GARCH(1,1) model:

Estimated parameters are shown in table 1.

Table 1: ARMA(1,1)-GARCH(1,1) for LPUSD series

Optimal Parameters				
	Estimate	Std.Error	t value	P-value
μ	0.00006	0.000010	5.8739	0.000000
$ar1$	0.65464	0.038831	16.8587	0.000000
$ma1$	-0.94893	0.009000	-105.4357	0.000000
c_0	0.00000	0.000000	2.3712	0.017732
α	0.15819	0.012694	12.4615	0.000000
β	0.84081	0.009641	87.2133	0.000000
Information Criteria ---------------------- Akaike -7.8868 Shibata -7.8868 Hannan-Quinn -7.8807 Bayes -7.8702				

The table 1 reveals that both coefficients related with ARMA(1,1) are significant and that also ARCH and GARCH terms are both significant. As $\alpha+\beta=0.999$ indicate that there is stationarity in volatility of LPUSD. The result is that we estimate a ARMA(1,1) -GARCH(1,1) and the $AIC=-7.8868$and $BIC=-7.8702$.

4.1.2 Estimation of ARMA (1,1)-GJRGARCH(1,1) model:

Table 2: ARMA (1,1)-GJRGARCH(1,1) for LPUSD series

Optimal Parameters				
	Estimate	Std.Error	t value	P-value
μ	0.000056	0.000042	1.34154	0.179745
$ar1$	0.644035	0.149061	4.32060	0.000016
$ma1$	-0.94819	0.028754	-32.97594	0.20865
c_0	0.000000	0.000002	0.20865	0.834721
α	0.152100	0.052139	2.91721	0.003532
β	0.810371	0.009641	10.73859	0.000000
γ	0.509323	0.050156	10.1547	0.00000
Information Criteria				

Akaike -7.8935			Shibata -7.8935	
Bayes -7.8742			Hannan-Quinn -7.8864	

Result from the table 2. Indicate that the estimated coefficient and both coefficient related to ARMA in the mean equation are significant, and both ARCH and GARCH terms are also significant. In ARMA(1,1) -GJRGARCH(1,1) and the $AIC = -7.8935$ and $BIC = -7.8742$.

4.1.3 Estimation of ARMA(1,1)-EGARCH(1,1) model:

Table 3: ARMA (1,1)-EGARCH(1,1) for LPUSD series

Optimal Parameters				
	Estimate	Std.Error	t value	P-value
μ	0.000119	0.000000	241.9614	0.00000
$ar1$	0.804518	0.008530	94.3138	0.00000
$ma1$	-0.98404	0.000040	-24708.43	0.00000
c_0	-0.62172	0.099166	-6.2695	0.00048
α	-0.04012	0.011491	-3.4914	0.00048
β	0.934724	0.010102	92.5300	0.00000
γ	0.509323	0.050156	10.1547	0.00000
Information Criteria				

Akaike -7.8483		Shibata -7.8483		
Hannan-Quinn -7.8412		Bayes -7.8290		

The table 3 reveals that both coefficients related with ARMA(1,1) are significant and that also ARCH and GARCH terms are both significant. The result is that we estimate a ARMA(1,1) -EGARCH(1,1) and the $AIC = -7.8483$ and $BIC = -7.8290$.

4.1.4 Estimation of ARMA(1,1)-EGARCH(1,1) model:

Table 4: ARMA (1,1)-APARCH(1,1) for LPUSD series

Optimal Parameters				
	Estimate	Std.Error	t value	P-value
μ	0.000113	0.000004	30.2714	0.00000
$ar1$	0.791639	0.011147	71.0180	0.00000
$ma1$	-0.979714	0.000300	-3267.7563	0.00000
c_0	0.000379	0.000044	8.6422	0.00000
α	0.436043	0.036202	12.0447	0.00000
β	0.650374	0.027535	23.6199	0.00000
γ	0.124666	0.028949	4.3064	1.7e-05
δ	1.00000	NA	NA	NA
Information Criteria -------------------------- Akaike -7.9022 Shibata -7.9023 Bayes -7.8829 Hannan-Quinn -7.8952				

Result from the table 4. Indicate that the estimated coefficient and both coefficient related to ARMA in the mean equation are significant, and both ARCH and GARCH terms are also significant. In ARMA(1,1)-APARCH(1,1) and the $AIC = -7.9022$ and $BIC = -7.8829$.

4.2.5 Comparing information criteria:

All the models are good as they have significant coefficients and terms, to compare them we use the Akaike information criterion (SIC) and Bayesian info criterion (BIC). The table 5 summarizes their AIC and BIC.

Table 5: information criterion for selected GARCH family models for LPUSD series

	ARMA-GARCH	ARMA-EGARCH	ARMA-GJRGARCH	ARMA-APARCH
Akaike	-7.886757	-7.848294	-7.893499	-7.902243
Bayes	-7.870206	-7.828984	-7.874189	-7.882933
Shibata	-7.886774	-7.848318	-7.893523	-7.902266
Hannan-Quinn	-7.880685	-7.841210	-7.886415	-7.895159

Conclusion: The table 5 reveals that the best model among these four models to be the ARMA(1,1) -APARCH(1,1) as is the one with the lowest information criteria.

The diagnostic checking of standardized squared residuals shows that residuals are not serially correlated and that there is no ARCH-effect in residuals, but residuals are not normally distributed and they have fat tail.

4.4. FITTING GENERALIZED PARETO DISTRIBUTION:

4.4.1 Selection of threshold:

The peak over threshold method uses a selected threshold, the set of exceedance above a threshold are said to follow the GPD (generalized pareto distribution)model, after which point and interval estimates for risk measures such as VaR(Value at Risk) and ES(Expected Shortfall) are calculated.

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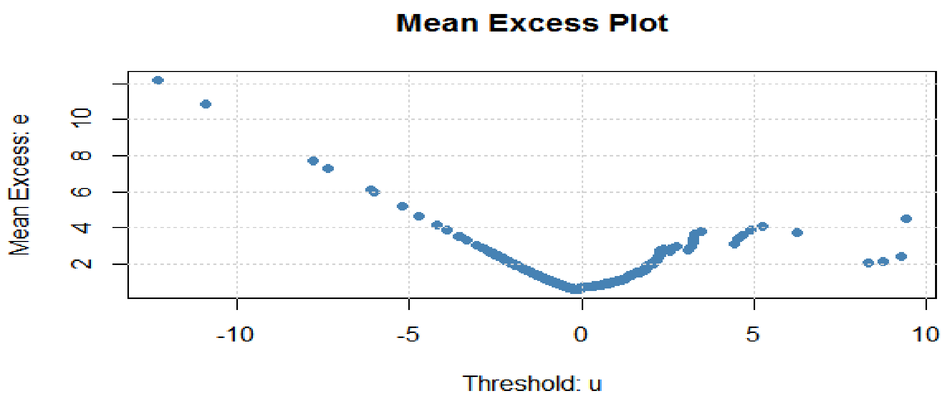

Figure 3: the mean excess plot for the ARMA(1,1)-APARCH(1,1) standardized residuals

Using results from R and considering plots the mean excess plot , we estimate a threshold of 1.36, and 102 data points above the threshold, about 5% of fitted data and which is large enough to make a good estimation.

4.4.2 GPD parameter estimation:

The estimated parameter of standardize residuals extracted from the ARMA(1,1)-APARCH(1,1) are shape and scale parameters, the shape parameter which is $\xi = 0.2079719$ which is always positive for financial data and the estimate of the scale parameter is $\sigma = 0.5403070$. Both shape and scale parameters are estimated using the maximum likelihood estimation. The estimated parameters are checked using the excess distribution shown in figure 4. Figure 4 show that the residuals over the selected threshold fit the GPD model. The fitting outcome of truncated $u = 1.36$

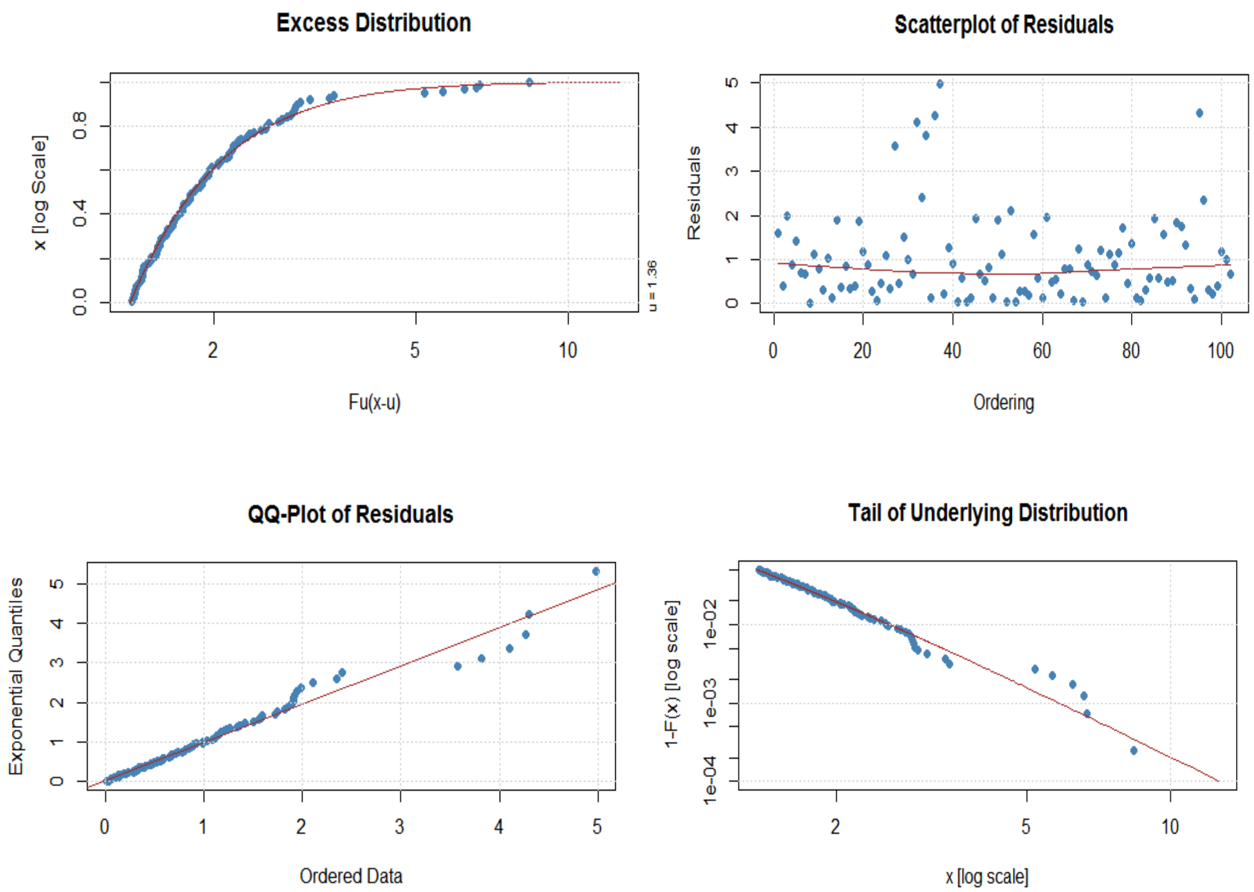

Figure 4: GPD fit plot for ARMA-APARCH residuals.

The Q-Q Plot for residuals (in Figure 4) is roughly a straight line of a unit slope passing through the origin. The QQ-Plot depicts points which are approximately linear, which shows that GPD is reasonably fit for the exceedance above threshold.

So based on the diagnostic plot these figures (figure 4) we can conclude that the model GPD (Generalized Pareto Distribution) fit well our series (residuals ARMA-APARCH). This means that GPD model can be used to estimate the two risk measures value at risk (VaR) and the Expected Shortfall (ES). The estimated VaR and ES are shown in table 8.

Table 6: estimation of VaR and Expected Shortfall with GPD

Probability	Value At Risk	Expected shortfall
0.900	0.9291945	1.659614
0.950	1.3520168	2.208401
0.975	1.8477571	2.851829
0.990	2.6366582	3.875755
0.995	3.3539416	4.806727
0.999	5.5332302	7.635256

Results in the table 12, shows the estimated values of VaR (Value At Risk) and the ES (Expected Shortfall) with different probabilities using the POT (Peak Over Threshold) method on residuals extracted from ARMA(1,1)-APARCH(1,1,1). Using the equation (2.25) and (2.26) the model for the VaR and ES are given by:

With the probability of 95%, the model for VaR and ES are given by:

$$VaR_q^t = \mu_{t+1} + 1.3520168\sigma_{t+1} \quad (24)$$

$$ES_q = \mu_{t+1} + 2.208401\sigma_{t+1} \quad (25)$$

With the probability of 99%, the model for VaR and ES are given by:

$$VaR_q^t = \mu_{t+1} + 1.3520168\sigma_{t+1} \quad (26)$$

$$ES_q = \mu_{t+1} + 2.208401\sigma_{t+1} \quad (27)$$

Where μ_{t+1} and σ_{t+1} are respectively the conditional ARMA-APARCH estimates of mean and volatility.

4.5 FITTING THE GENERALIZED EXTREME VALUES (GEV):

This study selects a quarterly block size to perform a block maxima method, and the maximum is selected in each block to finally fit them to a GEV. Figure 4 shows a GEV fit plots

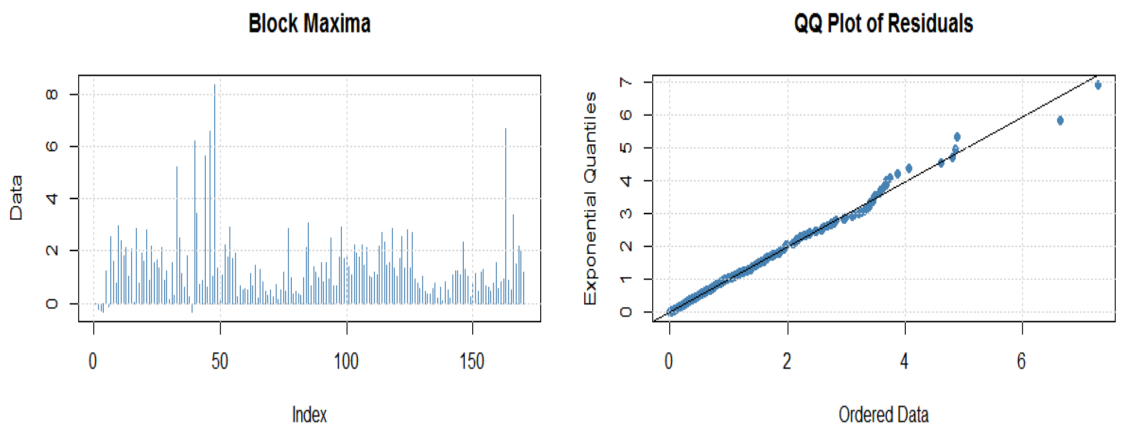

Figure 4: ***GEV fit plot for ARMA-APARCH residuals.***

The Q-Q plot of residuals in the figure 4 shows how residuals go with the straight line; this is a sign for fat tail behaviour, which shows that GEV is reasonably fit for the exceedance above threshold. The block maxima plot, gives the plot of excesses above 5% quantile, it plots the time series of residuals from ARMA-APARCH model for the tail. These two plots shows that our series well fit GEV model.

4.5.1 GEV parameter estimation and risk measures:

The output of the estimated parameter for GEV (Generalized Extreme Values) with R using the maximum likelihood estimator with a quarterly block show that the GEV model was fitted to the residuals extracted from ARMA(1,1)-APARCH(1,1,1), the estimated shape parameter is $\xi = 0.1435392$ and the scale parameter is estimated as $\sigma = 0.5746320$, the mean selected is $\mu = 0.3229868$. Since the shape parameter is positive, the distribution of data is of Fréchet-type (fat tail).

The risk measures value at risk is calculated using the equation (13), where the VaR is given by:

$$VaR(\alpha) = \mu + \frac{\sigma_t}{\xi}\{(-(\ln(\alpha))^{-\xi}) - 1\}$$

The estimated VaR with the probability of 95% and 99% are respectively 2.4513092 and 3.7446016

With the probability of 95%, the model for VaR is given by:

$$VaR_q^t = \mu_{t+1} + 2.4513092\sigma_{t+1}$$

With the probability of 99%, the model for VaR and ES are given by:

$$VaR_q^t = \mu_{t+1} + 3.7446016\sigma_{t+1}$$

5. CONCLUSION

On the basis of estimated VaR (Value At Risk) model and ES (Expected Shortfall) model using the selected best model among the GARCH-family models combine with EVT(Extreme Value Theory) applied on the Rwandan exchange market using the daily exchange rate (USD/RWF) data stating from 01/01/2010 up to 07/31/ 2015. This thesis has two main parts, the first part had as objective to choose the best model among the selected GARCH-family models that well fit our series; the second part had as objective to apply the EVT (Extreme Value Theory) approach on residuals of the selected model in part one using both extreme models GPD(Generalized Pareto Distribution) and GEV(Generalized Extreme Values)

This study takes us to the following conclusion: In part one:

1. The ARCH test in this study has confirm that there is ARCH effect in our series and provides evidences that ARCH and GARCH-family models can be applied to our series

2. .Among GARCH-family models, ARMA(1,1)-APARCH (1,1) is selected as the most suitable model for modeling the exchange market in Rwanda as it provides the lowest AIC and SIC values and significant coefficients, compared to other models.

3. The diagnostic residuals checking ARMA(1,1)-APARCH (1,1) shows no evidence for serial correlation in the squared residual, that there is no ARCH effect in squared residuals. The only problem with our model is that squared residuals are not normally distributed and have fat tail which is also good to be applied with EVT approach.

In part two:

1. The study first extracted residuals from ARMA(1,1)-APARCH(1,1) and fit them to a GPD model. Results show that the estimated parameter fit very well a GPD model and diagnostic plots made proved that GPD well fit our series. The study uses parameters from GPD model to estimate the value at risk and expected shortfall with different probabilities.

2. Secondly the study used the extracted residuals from ARMA(1,1)-APARCH(1,1) and fit them to a GEV model. Result shows again that the estimated parameter fit very well a GEV model and diagnostic plots made proved that GEV well fit our series. Finally we used estimated parameter from a GEV model estimate the value at risk with different probabilities.

3. Finally the estimated value at risk and expected shortfall from GPD and GEV are combined with the forecasted variance and mean from the ARMA-APARCH model to estimate the VaR and ES. The result shows that using GEV model the exchange market is exposed to high risk than using the GPD model.

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